Critical behavior of crisis-induced transition to spatiotemporal chaos in parameter space

Kaifen He
CCAST (World Laboratory), 8730 Beijing, 100080 China
and Institute of Low Energy Nuclear Physics, Beijing Normal University,
100875 Beijing, China

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In previous works we reported a transition mechanism from a temporal chaos (TC) to spatiotemporal chaos (STC) through a crisis due to a collision to a saddle steady wave (SSW). However, the transition also displays as a critical phenomenon in parameter space. In the present work the time variations of mode interaction energy, $\delta E_k^I(t)$, of the perturbation wave (PW) with its carrier SSW are calculated. In the TC state in all the dimensions the motion is dominated by negative $\delta E_k^I(t)$. With variation of the parameter in one dimension $\delta E_{k=1}^I(t)$ becomes smaller and smaller while statistically more balanced in its negative and positive values. The critical parameter point for the crisis is right at the place where the time-averaged negative and positive $\delta E_{k=1}^I(t)$ are equal. A power-law behavior is observed when approaching to the point. After the crisis in the STC state the motion with positive $\delta E_{k=1}^I(t)$ suddenly becomes much stronger than that with negative ones. In addition, it is shown that stable orbit of the SSW is a boundary of the PW motion, it behaves like a potential well that constrains the PW motion.

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I. INTRODUCTION

It is widely accepted that turbulence is a deterministic phenomenon, it can be investigated from the point of view of a dynamical system. However, as pointed out by M.C. Cross, considerable debate and confusion have arisen concerning the ''proper'' use of the word turbulence [1]. In nonlinear dynamics, usually this word is not limited in the traditional meaning in fluid dynamics, it has come to be used more widely, more particularly for so-called spatiotemporal chaos (STC). Indeed, one of the main purposes of studying STC is to reveal the underlying physics of turbulence, however, with different nonlinear models various types of STC can be observed [1], and obviously they can be attributed to different mechanisms. For instance, a transition to STC through intermittency has been observed, it is a continuous process that is much reminiscent of a second-order phase transition [2]; In three-dimensional Rayleigh-Bénard model, the authors find direct transition to STC, it is shown that the time-averaged structure factor has a scaling behavior near onset [3]. Other types of spatiotemporal disorders such as phase defects are also studied [4]. Recently we reported a transition to STC [5–7] with the following driven/damped drift-wave equation,

$$\frac{\partial \phi}{\partial t}+a\frac{\partial^3 \phi}{\partial t\partial x^2}+c\frac{\partial \phi}{\partial x}+f\phi\frac{\partial \phi}{\partial x}=-\gamma\phi-\epsilon\sin(x-\Omega t). \qquad (1)$$

In contrast to Ref. [2] in our case the transition is not a continuous process. It has been shown that onset of the transition is through a crisis [8] due to a collision to an unstable orbit of a saddle steady wave (SSW), it is likely a first-order phase transition. Another interesting point in the observation is a STC state after the transition shows a power-law spectrum, $\langle\phi^2\rangle\sim k^{-\beta}$ with $\beta\approx 5/2$. In comparison with the well-known Kolmogorov energy spectrum $\sim k^{-5/3}$ of fully-developed turbulence [9], one can find that it is a power law as well, but with different index. Therefore, it is reasonable to expect that exploring the transition mechanism of the crisis-induced STC displaying in Eq. (1) can be helpful for understanding the physics of a fully-developed turbulence.

In Ref. [5] it has been shown that for an occurrence of the crisis in the chosen parameters there must be a SSW solution, $\phi_0^*(x-\Omega t)$, here the subscript indicates a steady wave, superscript indicates a saddle type. By considering the dynamics of a PW on its carrier SSW we find that the collision of the PW chaotic attractor to an unstable orbit of the SSW is responsible for the crisis-induced transition to STC. On the other hand, the transition also displays as a critical phenomenon in parameter space. Simulation shows that it can occur only if the control parameters are beyond certain critical values. Our motivation in the present work is to clarify the physical implication of this critical phenomenon. To this end the wave energies of different wave number k are calculated. In Sec. II we compare the time variations of the PW mode energies before and after crisis in every dimension k, including the interaction energy between the PW and its carrier SSW (δE_k^I) as well as the PW self-energy (δE_k^S). In Sec. III the time averages of $\delta E_k^I(t)$ are calculated for its negative and positive values, respectively. It is found their relative intensity can be a characteristic quantity for studying the critical phenomenon. Furthermore, it is shown that the stable orbit of carrier SSW is a boundary of PW motion. Finally, Sec. IV is a discussion.

II. ENERGY OF PARTIAL WAVE BEFORE AND AFTER THE CRISIS

In Ref. [6,7] we studied the mechanism for a crisis-induced transition from temporal chaos (TC) to STC state, we have shown that a ''pattern resonance'' of the realized wave pattern with a SSW solution is responsible for triggering the crisis. That is, the existence of a SSW solution is a necessary condition for the occurrence of the transition. As

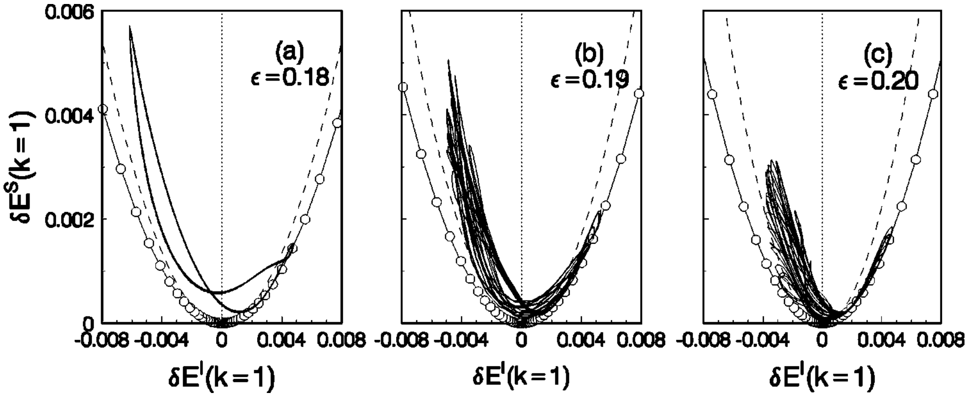

FIG. 1. Self-energy $\delta E_k^S(\tau)$ as a function of interaction energy $\delta E_k^I(\tau)$ of mode $k=1$ for $\Omega=0.65$, (a) $\epsilon=0.18$, (b) $\epsilon=0.19$, (c) $\epsilon=0.20$. All are in a spatially ordered state. The stable orbit (solid line with circles) and unstable orbit (dashed line) are also shown, respectively.

mentioned above, this transition also manifests as a critical phenomenon in parameter space. For a fixed Ω such transition can only be observed when ϵ is beyond a critical value ϵ_c, despite the fact that when $\epsilon<\epsilon_c$ a SSW solution may also exist. In other words, existence of a SSW solution does not necessarily cause a crisis, it is not a sufficient condition for the transition. There must be some additional physics that causes the critical phenomenon in parameter space.

To study the problem, a ''wave energy'' representation is convenient. The total ''wave energy'' in a period 2π of the system is defined as

$$E(t)=\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}\left[\phi^2-a\left(\frac{\partial\phi}{\partial x}\right)^2\right]dx. \tag{2}$$

Without driving ϵ and damping γ, $E(t)$ is a constant of motion of Eq. (1). In general, $E(t)$ can show behaviors such as constants, periodic, and different styles of chaotic motion. Since E depends time t only, its behaviors provide us a clue to classify, to a certain extent, the dynamical states of this space-time dependent system, and further to study the bifurcation mechanisms. However, for the present purpose this global quantity of the system is not sufficient. First of all, as mentioned above many complicated nonlinear behaviors can be explained on the hypothesis that they are the results of interaction of a SW with its PW. Therefore, energies of the SW and PW as well as their interaction should be studied. On the other hand, among the many infinite dimensions of this space-dependent system, not all the dimensions are critical in the onset of crisis-induced transition. Instead, e.g., for the example in Ref. [6], as a result of the mode-mode couplings only one dimension with the wavenumber $k=1$ becomes crucial. In this dimension the PW motion gets free from its carrier after the crisis, while the other PW partial waves are still trapped in the carrier SSW, respectively. For these reasons, in the following we calculate the wave energies in respect dimensions.

Since a SW of Eq. (1) has a form of $\phi_0(x-\Omega t)$, it is convenient to analyze the problem in the frame $\xi=x-\Omega t,\tau=t$. Let us set

$$\phi(\xi,t)=\phi_0(\xi)+\delta\phi(\xi,\tau), \tag{3}$$

and make the Fourier expansion for both $\phi_0(\xi)$ and $\delta\phi(\xi,\tau)$, $\phi_0(\xi)\equiv\Sigma_k\phi_{0,k}=\Sigma_kA_k\cos(k\xi+\theta_k)$, $\delta\phi(\xi,\tau)\equiv\Sigma_k\delta\phi_k(\tau)=\Sigma_kb_k(\tau)\cos[k\xi+\alpha_k(\tau)]$, then in every dimension, a mode energy is consisted of three parts: the self-energy of $\phi_{0,k}$:

$$E_{0,k}^S=\frac{1}{4}(1-ak^2)A_k^2, \tag{4}$$

the interaction energy between $\phi_{0,k}$ and $\delta\phi_k(\tau)$:

$$\delta E_k^I(\tau)=\frac{1}{2}(1-ak^2)A_kb_k(\tau)\cos[\theta_k-\alpha_k(\tau)], \tag{5}$$

and the self-energy of $\delta\phi_k(\tau)$:

$$\delta E_k^S(\tau)=\frac{1}{4}(1-ak^2)b_k^2(\tau). \tag{6}$$

In these expressions, $E_{0,k}^S$ is a constant of motion because A_k is a constant depending only on the control parameters $\{\Omega,\epsilon\}$ (the other parameters $a<0,c,f,\gamma$ are fixed); on the other hand $\delta E_k^S(\tau)$ depends only on the PW mode amplitude, while the interaction energy $\delta E_k^I(\tau)$ depends also on the phase difference $\theta_k-\alpha_k(\tau)$ between the modes of the SW and PW. In the (ξ,τ) frame the modes of SW, $\{A_k,\theta_k\}$, can be calculated from the steady mode equations of Eq. (1), $\partial\phi_0/\partial\tau=0$. With this SW solution the PW mode $\{b_k(\tau),\alpha_k(\tau)\}$ can then be computed from the mode equations of the following equation derived from Eq. (1),

$$\frac{\partial}{\partial\tau}\left[1+a\frac{\partial^2}{\partial\xi^2}\right]\delta\phi-\Omega\frac{\partial}{\partial\xi}\left[1+a\frac{\partial^2}{\partial\xi^2}\right]\delta\phi+c\frac{\partial}{\partial\xi}\delta\phi+\gamma\delta\phi$$
$$+f\frac{\partial}{\partial\xi}[\phi_0(\xi)\delta\phi]+f\delta\phi\frac{\partial}{\partial\xi}\delta\phi=0. \tag{7}$$

In the following we fix $\Omega=0.65$, and study the behaviors of the system motion when ϵ crosses the critical point $\epsilon_c\sim0.20$. In this parameter regime, a steady wave $\phi_0(\xi)$ is of saddle type, $\phi_0^*(\xi)$, the corresponding $E_0^*\equiv E(\phi_0^*)$ locates at the negative tangency slope of a hysteresis. By solving the mode equations of Eq. (7) $\{b_k,\alpha_k\}$ can be obtained, and hence obtained $\delta E_k^S(\tau)$ and $\delta E_k^I(\tau)$.

Figures 1 show some examples in which ϵ is near to but less than ϵ_c, with Fig. 1(a) $\epsilon=0.18$, Fig. 1(b) $\epsilon=19$, and

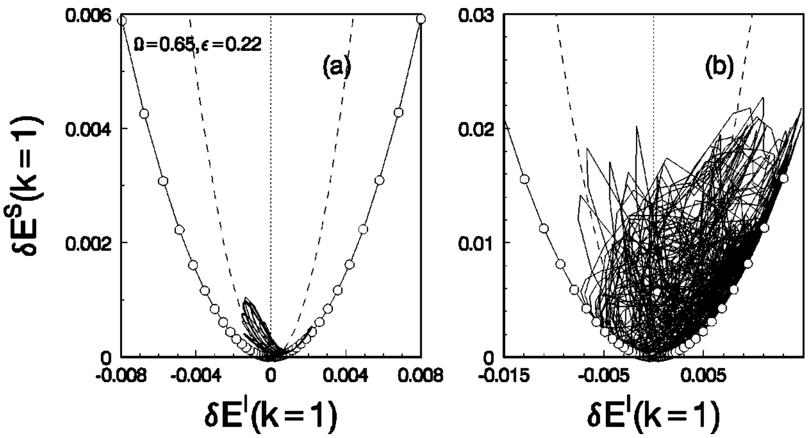

FIG. 2. Self-energy $\delta E_k^S(\tau)$ as a function of interaction energy $\delta E_k^I(\tau)$ of mode $k=1$ for $\Omega=0.65$, $\epsilon=0.22$. (a) transient TC state before the crisis, (b) asymptotic STC state after the crisis. The SO and UO are also shown.

Fig. 1(c) $\epsilon=20$. In all these cases no crisis happens, and their asymptotic motion is spatially smooth, despite that the temporal motion can be chaotic. In the plots the self-energies $\delta E_k^S(\tau)$ as functions of the interaction energies $\delta E_k^I(\tau)$ for $k=1$ are drawn, in Fig. 1(a) the motion is periodic, in Figs. 1(b) and 1(c) they are temporally chaotic. The stable orbit (SO, solid line with circles) as well as the unstable orbit (UO, dashed line) of the carrier SSW are also drawn respectively in the corresponding plots, they are calculated by neglecting the last term in the left-hand side of Eq. (7). As can be seen in their definitions a self-energy $\delta E_{k=1}^S(\tau)$ is positive-definite (because $a<0$ in physics), but an interaction energy $\delta E_k^I(\tau)$ can be negative or positive depending on the relative phase $(\theta_k-\alpha_k)$. One can see in Figs. 1 that an orbit makes excursion to both sides of $\delta E_k^I(\tau)=0$, and an interesting phenomenon is that in all the plots the center of trajectory is in the negative side. Furthermore, comparing Fig. 1(c) with Figs. 1(a) and 1(b) one can see that the orbits of $\delta E_{k=1}^S(\tau)$ vs $\delta E_{k=1}^I(\tau)$ are confined to a smaller and smaller range, in particular in the negative side, that is, the motion looks more balanced among the two sides when ϵ approaching to the critical point ϵ_c.

Figures 2 give $\delta E_k^S(\tau)$ vs $\delta E_k^I(\tau)$ for $k=1$ with $\Omega=0.65,\epsilon=0.22$. In this case a crisis-induced transition to STC can occur, so both the transient TC period before the crisis [Fig. 2(a)] and the asymptotic STC state after the crisis [Fig. 2(b)] are given, respectively. The SO and UO are also drawn. One can see that before the crisis in Fig. 2(a) both $\delta E_{k=1}^S(\tau)$ and $\delta E_{k=1}^I(\tau)$ are very small. In contrast, after the crisis in Fig. 2(b) the trajectory looks extremely chaotic that is wandering in a much larger phase space. It is very interesting that after the crisis obviously the main part of the trajectory shifts to the right-hand side where interaction energy $\delta E_{k=1}^I(\tau)$ is positive. Furthermore, although the orbit makes random walks, it is clearly bounded by the SO of the SSW. If looking at the plot in detail, one can find that sometimes the orbit seems to be reflected by the SO boundary, just like an elastic ball hitting on it.

Figures 3 show $\delta E_k^S(\tau)$ vs $\delta E_k^I(\tau)$ for the same parameters as Figs. 2, but with $k=2$, where Fig. 3(a) is before the crisis and Fig. 3(b) after the crisis. One can see that the motion looks much more chaotic after the crisis than before it. However, different from in Figs. 2, here either before or after the crisis, the orbit almost always travels in the negative side of $\delta E_{k=2}^I(\tau)$. We also notice that, like in the case of $k=1$, in Figs. 3 the SO are about the boundary of the PW motion. However in this case the orbit is occasionally lower than the SO slightly, it is not clear yet whether it comes from computational error.

III. CRITICAL PHENOMENON IN PARAMETER SPACE OF $k=1$ MODE AS A TIME-AVERAGED BEHAVIOR

In the last section we investigate motion of the modes before and beyond a critical point for the transition. We find that beyond the critical point after the crisis all the modes

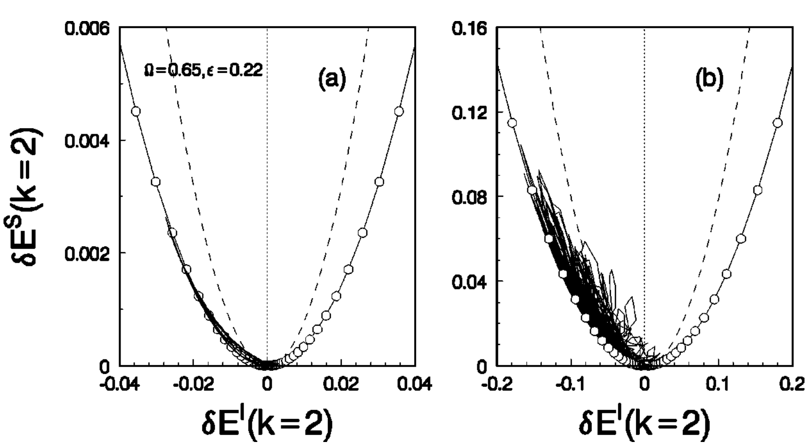

FIG. 3. $\delta E_k^S(\tau)$ versus $\delta E_k^I(\tau)$ of mode $k=2$, the same parameters as in Figs. 2. (a) before the crisis, (b) after the crisis. The SO and UO are also shown.

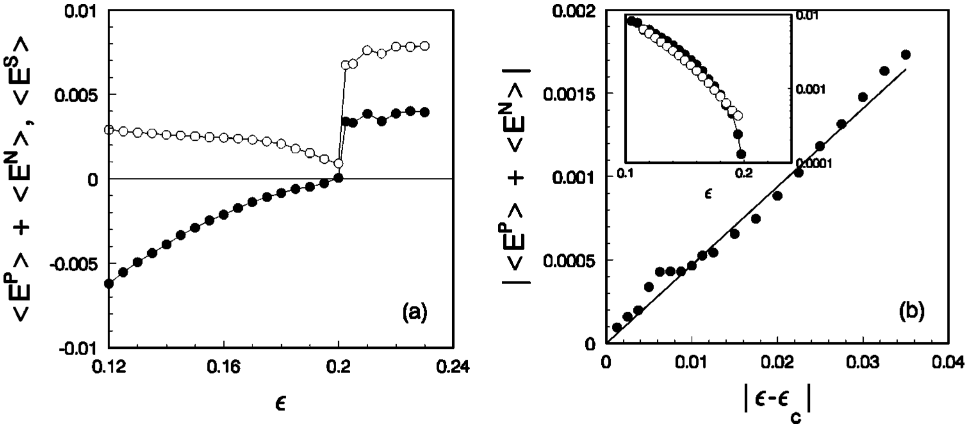

FIG. 4. (a) Variation of $\langle \delta E^P_{k=1}(\tau)\rangle + \langle \delta E^N_{k=1}(\tau)\rangle$ with ϵ (bullet) (the open circles give $\langle \delta E^S_{k=1}(\tau)\rangle$), $\Omega = 0.65$. (b) Its power-law behavior when approaching to $\epsilon - \epsilon_c$. The insert gives $|\langle \delta E^P_{k=1}(\tau)\rangle + \langle \delta E^N_{k=1}(\tau)\rangle|$ (bullet) and $|\langle \delta E^I_{k=1}(\tau)\rangle|$ (circle) varying with ϵ in semi-logarithmic plot, one can see that the simple average $\langle \delta E^I_{k=1}(\tau)\rangle$ has no trend to reach zero at ϵ_c.

behave very chaotic, however, on average only $k=1$ mode shows a remarkable change, i.e., the main part of the motion of $\delta E^I_{k=1}(\tau)$ is shifted to the positive side, in contrast to before the crisis where $\delta E^I_{k=1}(\tau)$ is mainly in the negative domain. This result suggests an investigation for the average behaviors of the motion. In this context we notice that Ref. [3] also pointed out that time-averaged quantity can be useful in studying a transition to turbulence. However, instead of the global one, here we study the time-averaged motion in different dimensions. In particular, we make the averages of $\delta E^I_k(\tau)$ respectively for their positive and negative values, they are

$$\langle \delta E^P_k(\tau)\rangle \equiv \frac{1}{T_P} \sum_{\delta E^I_k > 0} \delta E^I_k(\tau) \triangle \tau,$$

$$\langle \delta E^N_k(\tau)\rangle \equiv \frac{1}{T_N} \sum_{\delta E^I_k < 0} \delta E^I_k(\tau) \triangle \tau,$$

respectively, here $\triangle \tau$ is the time step, T_P and T_N are the time duration for the motion with positive and negative $\delta E^I_k(\tau)$ respectively in the computation.

In Fig. 4(a) the summation $\langle \delta E^P_k(\tau)\rangle + \langle \delta E^N_k(\tau)\rangle$ for mode $k=1$ is displayed as a function of ϵ (bullet). It can be seen in the plot that, when approaching to the critical point $\epsilon_c \sim 0.20$, $\langle \delta E^P_{k=1}(\tau)\rangle + \langle \delta E^N_{k=1}(\tau)\rangle$ approaches to zero from the negative side. After crossing the critical point when $\epsilon > \epsilon_c$ corresponding to the asymptotic STC states $\langle \delta E^P_{k=1}(\tau)\rangle + \langle \delta E^N_{k=1}(\tau)\rangle$ transits to big positive values. In the plot $\langle \delta E^S_{k=1}\rangle$ is also shown (open circle). From the result it is found that the critical transition point ϵ_c is right the position where the interaction energy $\delta E^I_k(\tau)$ has same averaged intensities for its negative and positive values. This is in agreement with what we have observed in Figs. 1(a)–1(c). In contrast, if making the average $\langle \delta E^I_k(\tau)\rangle$ as a whole (without separating the negative and positive values), although it becomes very small when near to the critical point ϵ_c, it does not really show the tendency of crossing zero at it. The inset in Fig. 4(b) gives a semi-logarithmic plot for the variations of $|\langle \delta E^P_{k=1}(\tau)\rangle + \langle \delta E^N_{k=1}(\tau)\rangle|$ (bullet) and $|\langle \delta E^I_{k=1}(\tau)\rangle|$ (open circle) with ϵ, from which we are convinced that it is the former but not the latter that crosses zero at ϵ_c.

In Fig. 4(b) we give the absolute value of $\langle \delta E^P_{k=1}(\tau)\rangle + \langle \delta E^N_{k=1}(\tau)\rangle$ as a function of $|\epsilon - \epsilon_c|$. Each point is ob-

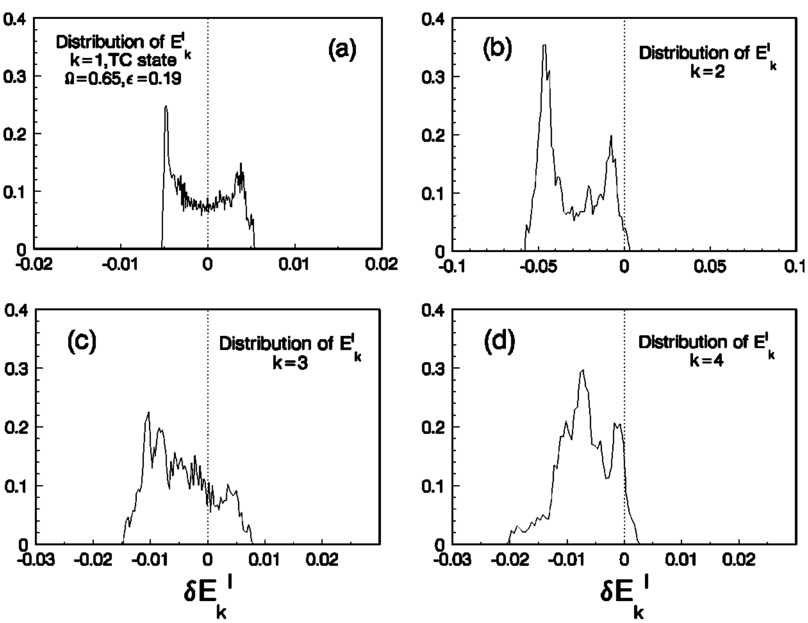

FIG. 5. Distributions of $\delta E^I_k(\tau)$ for $\Omega = 0.65$, $\epsilon = 0.19$. TC state. (a) $k=1$, (b) $k=2$, (c) $k=3$, (d) $k=4$.

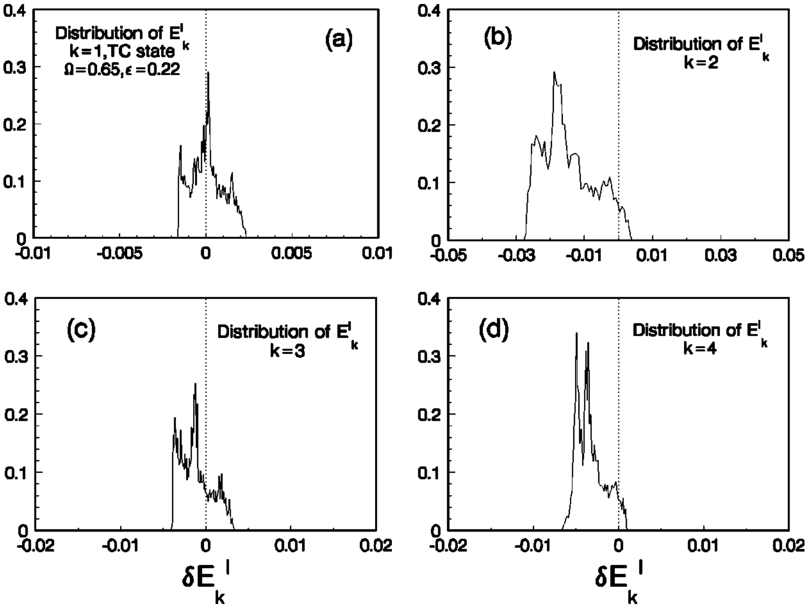

FIG. 6. Distributions of $\delta E^I_k(\tau)$ for $\Omega = 0.65$, $\epsilon = 0.22$. Transient TC state. (a) $k=1$, (b) $k=2$, (c) $k=3$, (d) $k=4$.

tained by averaging the results of ten runs in the attractor. It has a power-law behavior when approaching to the critical point, $|\langle \delta E^P_{k=1}(\tau)\rangle + \langle \delta E^N_{k=1}(\tau)\rangle| \sim g|\epsilon-\epsilon_c|$, here $g \approx 0.047$.

To compare the time-averaged motion of different modes the distributions of interaction energies $\delta E^I_k(\tau)$ are calculated respectively for $\epsilon<\epsilon_c$ and $\epsilon>\epsilon_c$. Figures 5 show the results for modes $k=1-4$ with $\epsilon=0.19<\epsilon_c$. In this parameter the asymptotic motion is a TC state. One can see that all the distributions of modes $k=1-4$ are peaked in the negative domain, and hence the time averages of the negative values are stronger.

When $\epsilon>\epsilon_c$ the asymptotic motion is in STC state, in this case the distributions of $\delta E^I_k(\tau)$ both before and after the crisis are calculated. For $\epsilon=0.22$ the results of $k=1-4$ are shown in Figs. 6 for the transient period before the crisis, while in Figs. 7 for the asymptotic motion after the crisis. Comparing Figs. 7 with Figs. 6 one can find a remarkable change occurring in mode $k=1$. Before the crisis in Fig. 6(a) the distribution of $\delta E^I_{k=1}(\tau)$ is extremely narrow, and it has a peak very near to zero but shifted slightly to the positive side. The area occupied by the positive one increases to about 53% of the total distribution, compared to 45% in Fig. 5(a); While after the crisis in Fig. 7(a) the distribution becomes wider with the peak further shifted to the positive side, the area in the positive side increases to 79%. The motion is now dominated by positive interaction energy.

In contrast, for $k\neq 1$, either before or after the crisis the distributions of $\delta E^I_k(\tau)$ are peaked at the negative domain [Figs. 6 and 7(b)–7(d)], they all have stronger averages for negative $\delta E^I_k(\tau)$. With increasing ϵ for all the modes the distribution shape changes very little in STC state. For example, for $\epsilon=0.205$, 0.210, 0.215, 0.220, the proportion of area with positive $\delta E^I_{k=2}(\tau)$ is about 0.017, 0.013, 0.026,

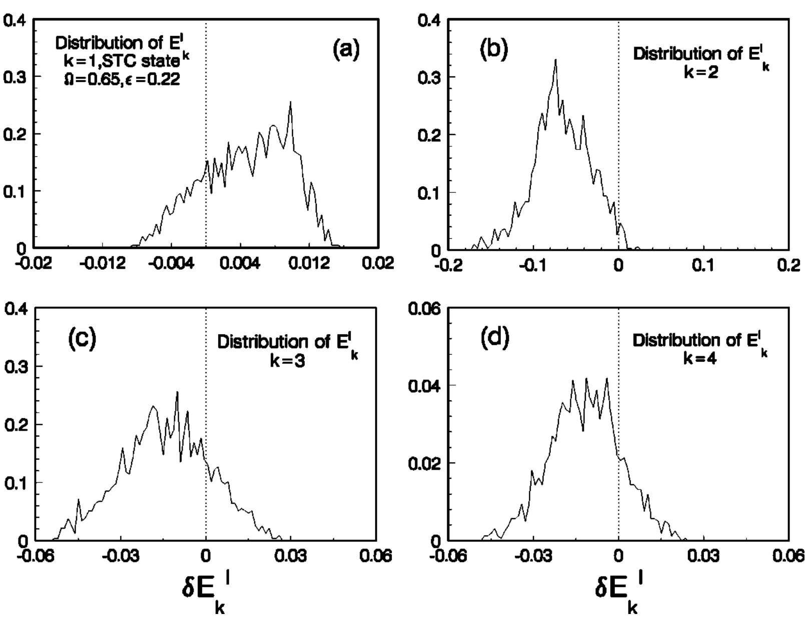

FIG. 7. Distributions of $\delta E^I_k(\tau)$ for $\Omega = 0.65$, $\epsilon = 0.22$. Asymptotic STC state. (a) $k=1$, (b) $k=2$, (c) $k=3$, (d) $k=4$.

0.017, respectively. That is, for $k \neq 1$ modes no tendency of increasing positive interaction energy with ϵ is observed.

The above results indicate that whether a crisis-induced transition to STC can occur is not only determined by the existence of a SSW solution, but also determined by the situation of the interaction between the PW and the SSW. If for all the modes the motion is dominated by negative $\delta E_k^I(\tau)$, the whole state displays as a lamina-like one. Only when approaching to a critical parameter point where, in a crucial dimension $k=1$, the averaged positive interaction energy $\langle \delta E_k^P(\tau) \rangle$ becomes equal to the negative one, $\langle \delta E_k^N(\tau) \rangle$, a crisis may occur. After the crisis, $\langle \delta E_{k=1}^P(\tau) \rangle$ gets much stronger than $\langle \delta E_{k=1}^N(\tau) \rangle$, while in the other dimensions still the negative ones are stronger.

IV. CONCLUSION AND DISCUSSION

In the present work the critical phenomenon in parameter space for the crisis-induced transition to STC is investigated. In the TC state in all the dimensions the PW motion has a tendency to have negative interaction energy with the carrier SSW. At the critical transition point for $k=1$ mode the motion with positive interaction energy gets balanced with the negative one. This critical phenomenon displays as a power-law behavior. We also show that stable orbit of the carrier SSW is a boundary of the chaotic trajectory of PW even after the crisis, it behaves like a potential well that constrains the PW motion.

This result is consistent with our observation that when in the TC state the motion of all the PW partial waves are well trapped in the trough of the respective carrier SSW, while after the crisis the $k=1$ PW partial wave gets free from it. To see this fact let us notice that $\delta E_k^I(\tau)$ depends on the phase difference $\alpha_k(\tau)-\theta_k$, that is, it takes a negative/positive value if PW partial wave is, roughly speaking, anti/in phase with that of the SSW. This agrees with what we have seen in the well-trapped/free state of the $k=1$ partial wave before/after the crisis, respectively (see Figs. 6(a) and 6(b) in Ref. [6]).

For the present example, mode $k=1$ plays a crucial role both in the critical phenomenon in parameter space as discussed above and in the critical transition in time evolution discussed in Ref. [6]. For $k \neq 1$ modes, we did not see any tendency to get free from their trapped states (or the relevant sign change of the characteristic quantity). A question is whether this phenomenon is a special case. Concerning the problem we should like to point out that $k=1$ is the slowest variable in the motion, it can slave the motion of other modes, therefore it is not surprising that $k=1$ plays the crucial role, its qualitative change as a result of mode couplings brings the global motion to a new state.

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